TM No. 881110

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Technical Memorandum

NOISE GENERATION IN CLASS A-D AMPLIFIERS

Date: 5 August 1988

Prepared by: R. J. Nielsen
R. J. Nielsen

ABSTRACT

The low frequency noise produced in generating a class A-D signal is studied. Noise levels are calculated using both computer models and analytic techniques. Although the treatment is not rigorous both methods show excellent agreement. The results indicate that the low frequency noise level is highly sensitive to the phase (time) jitter on the A-D amplifier output signal.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

There has been much discussion concerning the low frequency noise which is generated by class A-D amplifiers. The questions concern the probable cause and the resulting level of that noise. This study looks at some of the aspects of that noise from both a modeling and analytic approach. Although the approach presented here is not rigorous, it does provide insight and bounds to the problem.

BACKGROUND

A class A-D amplifier operates very much like pulse width modulation. A description follows: Consider a square wave output signal of approximately 25 kHz (switching frequency). This signal is the waveform to be modulated by the input signal. With no signal input (0% modulation or modulation index [MI]=0) the output waveform is a square wave oscillating at the switching frequency. The modulation process is linear. That is, the width of each positive pulse in the output signal is linearly related to the amplitude of the input signal. The linear transfer function is adjusted so that 100 percent modulation (MI=1) will not overdrive the output signal. Thus maximum input signal will correspond to a pulse width less than the period of the switching frequency and the minimum input signal corresponds to a pulse width greater than zero. In all cases the width of the pulse is determined and centered about a time corresponding to the center of the positive pulse in the switching waveform.

An A-D amplifier output signal which results when the input signal is a 1 kHz sinusoidal waveform is shown in Figure 1. The power spectrum of this signal is shown, using several frequency scales, in Figures 2-5. Much more detail, concerning these spectra, will be discussed in the next section.

At first glance it is tempting to note that the noise level, relative to the 1 kHz input signal, is in fairly close agreement with many measurements made on existing A-D amplifier systems. The results of the analysis will show, however, that this conclusion is incorrect. This low frequency noise will be shown to be an artifact of the modeling process while a significant portion of the real output noise can be attributed to jitter of the output signal.

ANALYSIS

The first approach to this problem was to model this signal on a digital computer, take an FFT of the resulting waveform, compute the power spectrum and note the resulting noise level in the low frequency range of the spectrum. Although appealing in its simplicity, there are several problems with this method. These difficulties can, however, be overcome if one is careful setting up the model and interpreting the results.

The digital parameters were selected so that the frequency components of interest would fall in bin centers. Also, to prevent skew in a sample square wave, the number of samples in one period of the switching frequency was set equal to a power of two. These criteria resulted in a sampling frequency of 1,638,400 Hz, a switching frequency of 25,600 Hz and a signal frequency of 1000 Hz. This resulted in 64 samples per cycle of the switching frequency. Other values could have been used but these numbers were close to the parameter values of interest. Using an 8192 point transform the resulting bin width was 200 Hz. In all cases 5 FFT's were averaged and no windows were used.

In spite of the high sampling frequency one still has to be careful about the aliasing which will occur in the resulting spectrum. This is demonstrated in Figures 6 and 7. Figure 6 is a time series of the unmodulated output signal (i.e., a square wave at 25,600 Hz). Figure 7 is a plot of the resulting spectrum. As you would expect the spectrum contains the fundamental frequency and all the odd harmonics. Note that the harmonics fall off very slowly and they remain high out to the Nyquist frequency. In fact, all of these frequency components are corrupted by aliasing. Fortunately, the lower frequencies are relatively unaffected by the foldover. Since our main interest is in the low level noise below 1000 Hz, we must still be careful that some high level frequency components do not get folded over into that frequency range.

Referring to the spectra of the class A-D amplifier (Figures 2-5) we note the presence of many discrete frequency components. There are components representing the signal frequency, the switching frequency, sidebands about the switching frequency, harmonics of these sidebands, etc. These frequency components are real and can be accounted for mathematically (Reference 1). The mathematics does not, however, show that any broadband noise is produced by the modulation process.

Our main interest is in the value of the noise below 1 kHz. Applying a bandwidth correction of 23 dB (10 log 200), the power spectra show noise level approximately 100 dB below peak signal level in this frequency range. This noise is not real and is a result of the sampling process. Recall that there are 64 discrete samples per cycle of the switching frequency. Thus, in the modulation process, the width of the positive pulse can only be set in increments. This results in a form of quantization error (quantization noise) which appear in the spectra of the class A-D amplifier.

The foregoing analysis does, however, give us a clue as to one of the real, and perhaps major, causes of noise in these amplifiers. In a broad sense, the quantization of the output signal can be said to be similar to the superposition of jitter on the vertical edges of the output signal.

Stability and noise problems will produce jitter in the output signal of a class A-D amplifier. Other forms of distortion of the output signal (overshoot, etc.) produce additional noise which will not be considered here.

An analytic approach to evaluate the low frequency noise as a function of the magnitude of the jitter was pursued. It was felt that the A-D amplifier jitter noise would not be very different from the noise resulting from the superposition of jitter on an unmodulated output signal (square wave). This assumption is verified, in a non-rigorous manner, by the comparison of model and analytic results. This also has the added benefit of greatly simplifying the mathematics which are detailed in Appendix I.

Each positive pulse in the output signal (square wave) had a nominal width of 1/(25600*2) seconds which is equivalent to 32 discrete samples. The pulse width was altered by the addition of an independent Gaussian random variable having a zero mean and a standard deviation sigma. Sigma can be thought of in terms of time (seconds) or the equivalent number of sample pulses (Appendix I). The assumption of a Gaussian distribution seems reasonable but obviously will not be valid for large values of sigma where overload (overlap of pulses) would occur.

Referring to Appendix I, the resultant spectrum (not including the discrete frequency components) is given by Equation 1.

$$S(f) = \frac{A^2 f_w}{\pi^2 f^2} \left\{ \left[1 - e^{-(2\pi f \sigma_\delta)^2} \right] \left[1 + \cos\left(\frac{\pi f}{f_w}\right) e^{-(2\pi f \sigma_\delta)^2} \right] \right\} \tag{1}$$

A plot of this spectrum is shown in Figures 8 and 9 for standard deviations corresponding to .2, 1, 2, 5 and 10 sample pulses. Using approximations, which are valid in the low frequency range of interest, this equation can be reduced to the following simple result (Equation 2).

$$S(f) \approx 8\, A^2\, f_w\, \sigma_\delta^2 \tag{2}$$

Equation 2 is plotted in Figure 10. This plot shows the low frequency noise level to be very sensitive to the magnitude of the jitter superimposed on the square wave. For example, a low frequency noise level 100 dB below the signal level would require a jitter standard deviation of .0128 samples or .0078 microseconds.

Effort was made to compare the analytic results with a computer model. To do this an unmodulated class A-D output signal (square wave) was altered by the addition Gaussian jitter. The width of the positive pulses was determined by selecting the discrete pulse width which was closest to the desired pulse width. This should work well except for large standard deviations where the pulses would overlap or small standard deviations where the pulse width would not change.

A comparison of the analytic results and the computer model are shown in Figures 11-15. Except in the extreme cases (sigma=.2 and sigma=10 pulses) the agreement is excellent.

The computer model was used in the same manner to generate a 100 percent modulated class A-D output signal containing Gaussian jitter. It was felt that the low frequency noise generated in the modulated case would not be much different than that generated in the unmodulated case. This jitter would be confounded with the jitter produced when sigma=0 (Figures 2-5). The noise produced when jitter is deliberately added is, however, significantly above the sigma=0 case so as to have negligible effect on the noise level. These results, and a comparison with the analytic model, are shown in Figures 16-20. Again the agreement is very good.

The approximate modulated low frequency noise levels are plotted in Figure 10 along with the unmodulated and analytic results. As expected the differences are slight and the analytic results appear to form a close upper bound to the low frequency noise levels. The discrepancy for small values of sigma is due to the computer model inability to approximate a Gaussian distribution in this region.

SUMMARY

This study attempts to account for some of the low frequency noise produced by class A-D amplifiers. The analysis shows the noise to be very sensitive to jitter on the output signal. Other sources of noise such as output signal overshoot, input signal distortion and system noise should also be considered.

The treatment is not complete and a rigorous solution to jitter on a fully modulated class A-D signal should be pursued. These results do, however, show good agreement between modeled and analytic approaches. The simple expression developed for low frequency noise should provide a useful guide in approximating low frequency noise levels in class A-D amplifiers.

REFERENCES

1. "Reference Data for Radio Engineers, 5th Edition", Howard W. Sams and Co., Inc., New York, 1970, ch. 21, pp. 18-19.

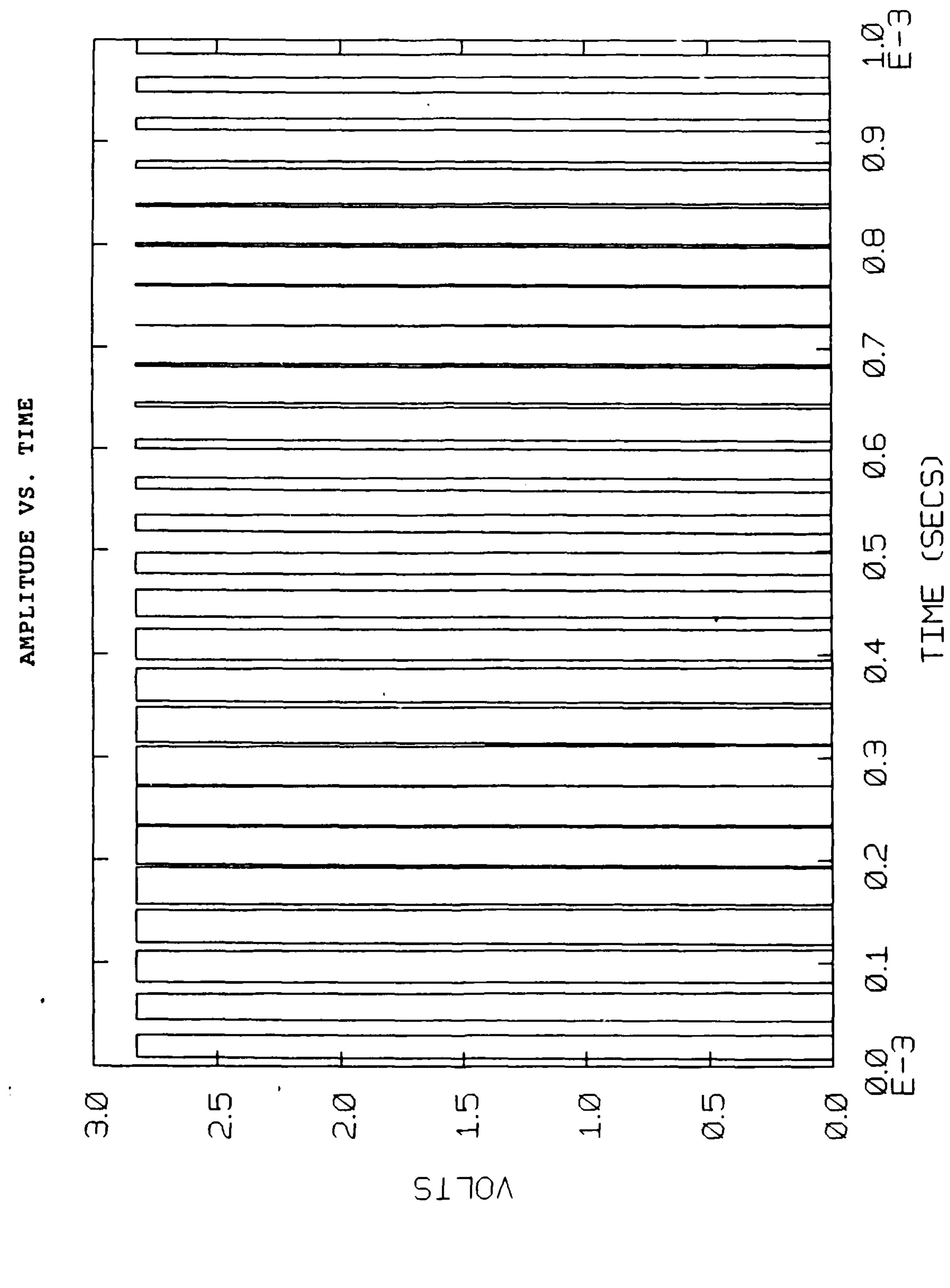

AMPLITUDE VS. TIME
TIME (SECS)
VOLTS
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.1
0.2
0.3
0.4
0.6
0.7
0.8
0.9
E-3

FIGURE 1

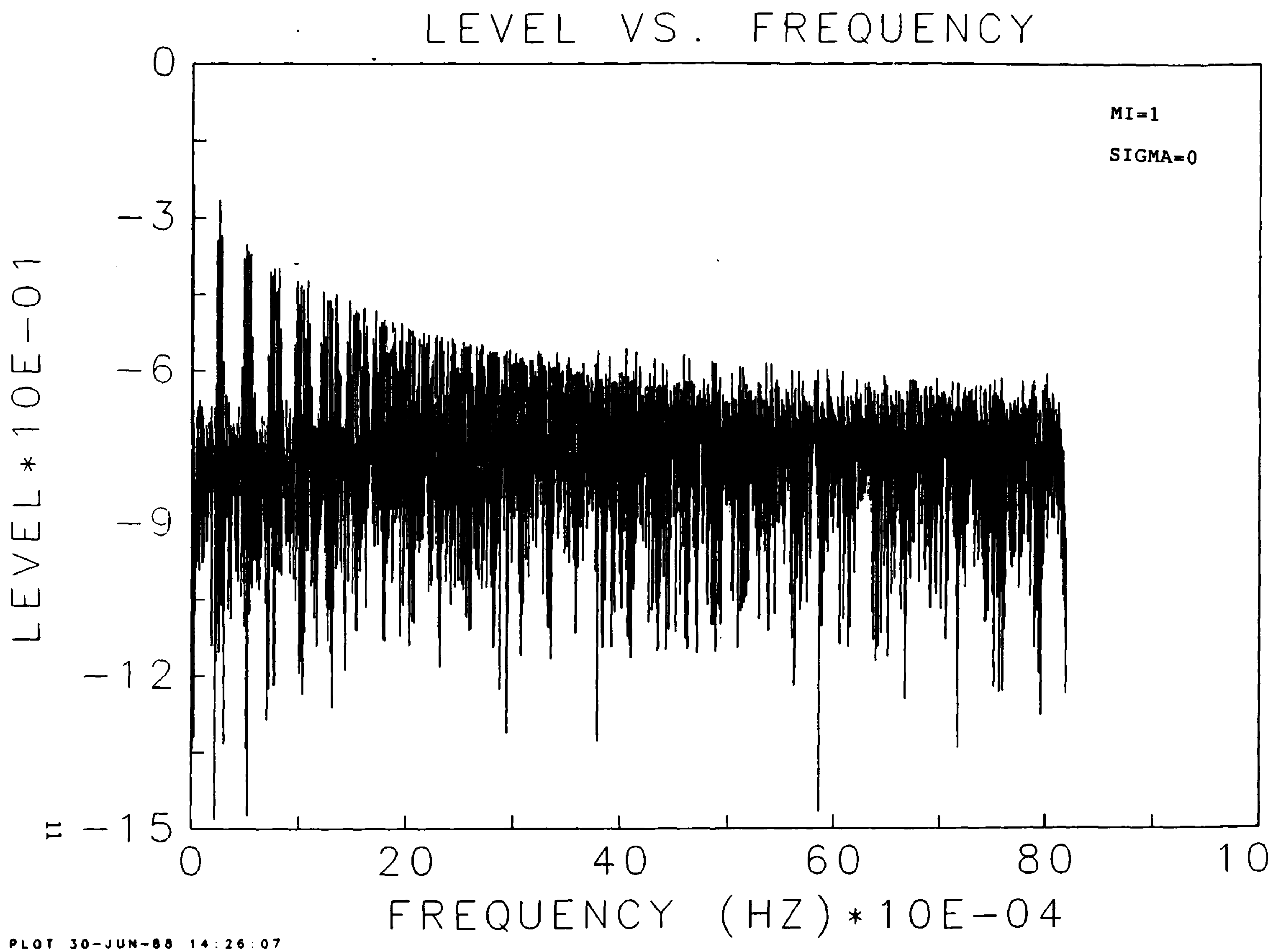

LEVEL VS. FREQUENCY
MI=1
SIGMA=0
0
-3
-6
-9
-12
-15
LEVEL*10E-01
0
20
40
60
80
100
FREQUENCY (HZ)*10E-04
PLOT 30-JUN-88 14:26:07

FIGURE 2

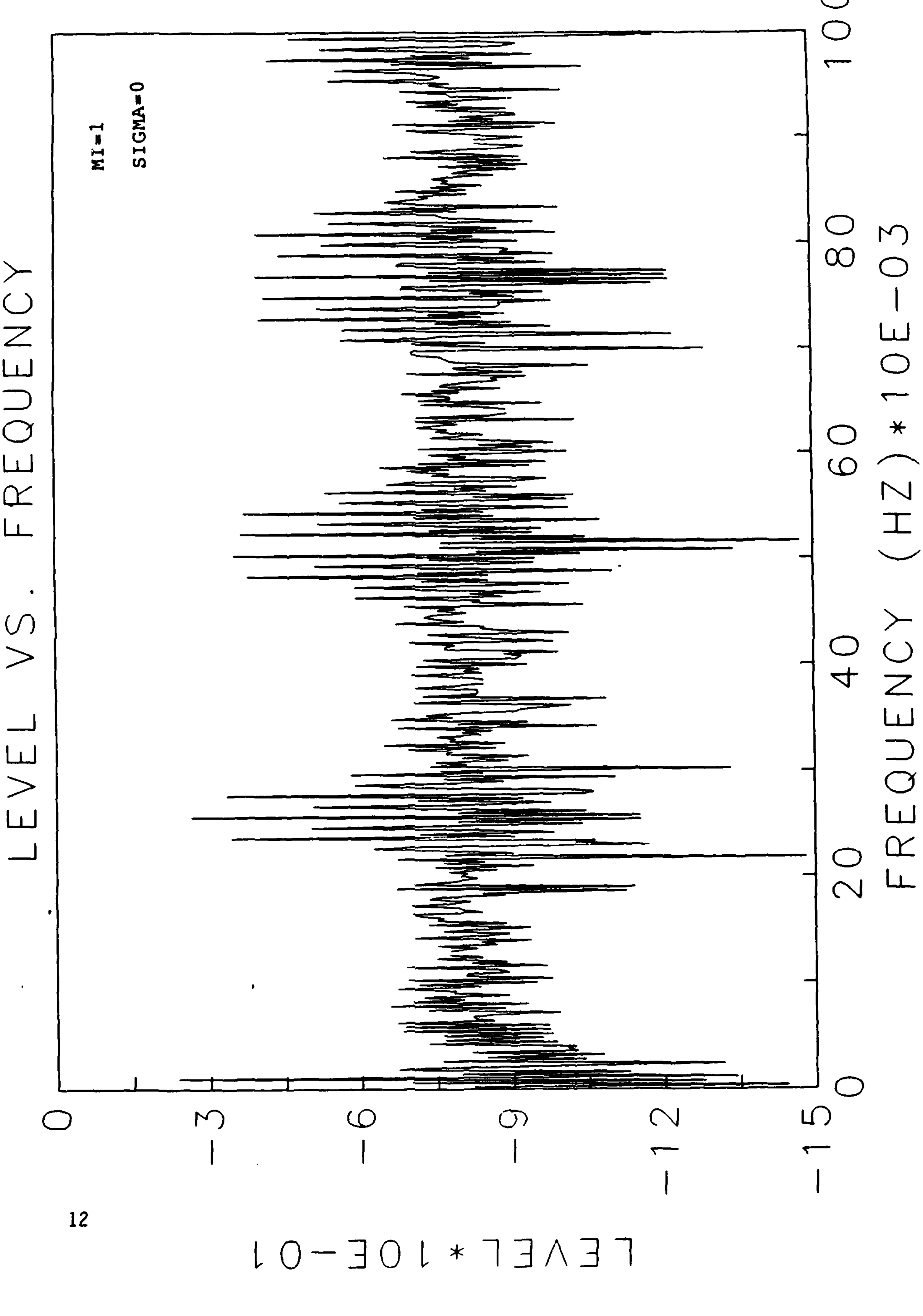


FIGURE 3

PLOT 30-JUN-88 14:25:39

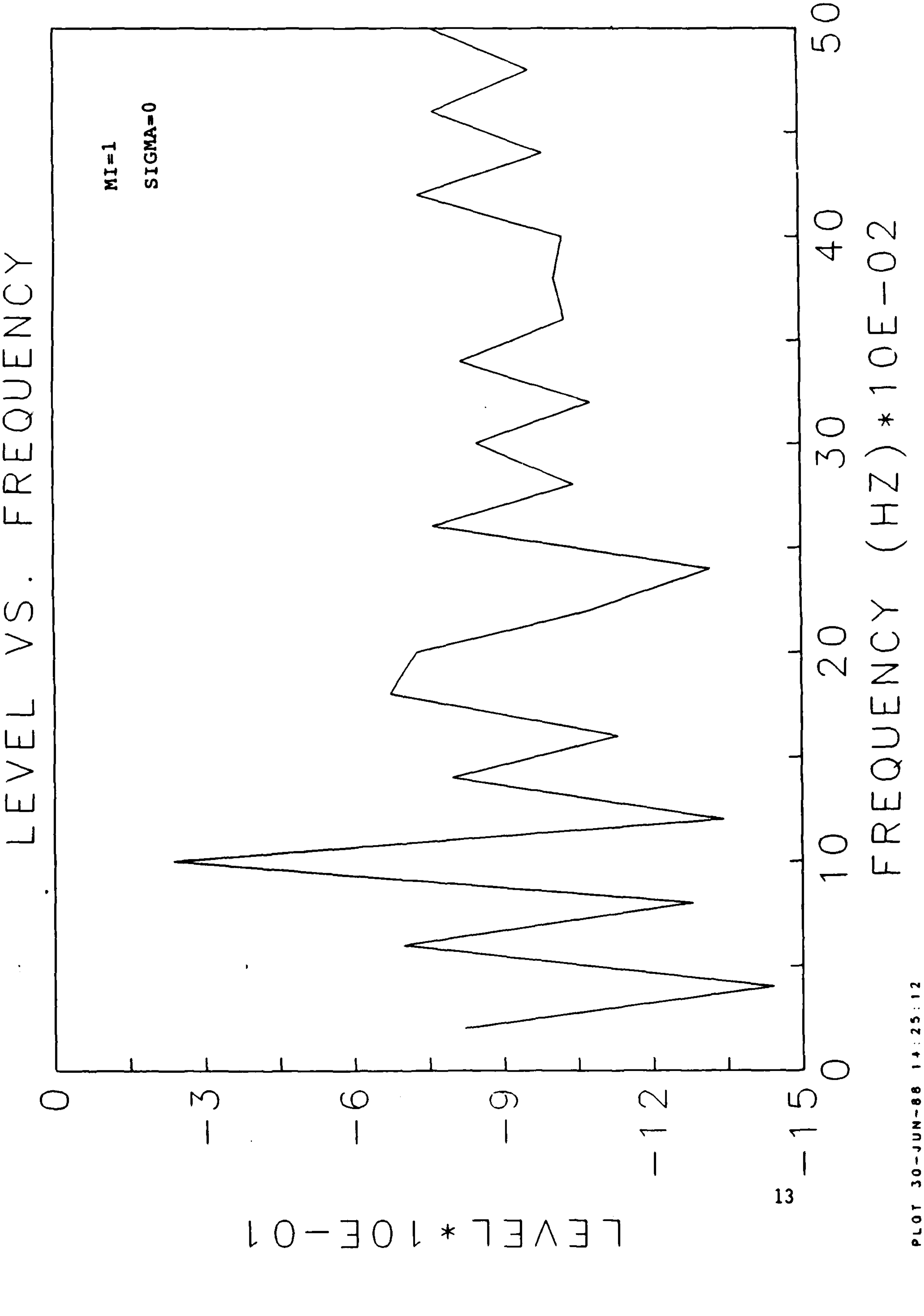


FIGURE 4

PLOT 30-JUN-88 14:25:12

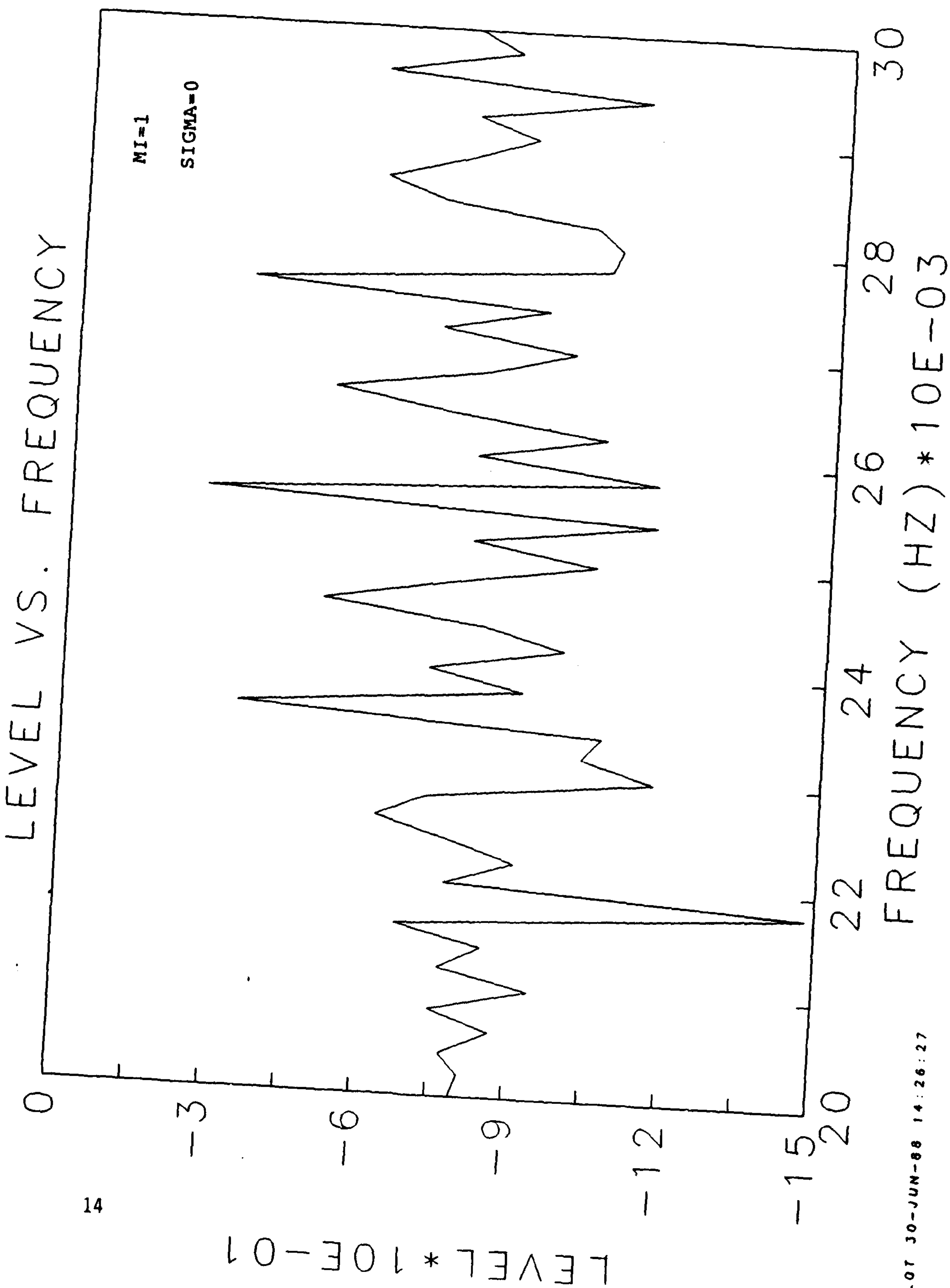


FIGURE 5

PLOT 30-JUN-88 14:26:27

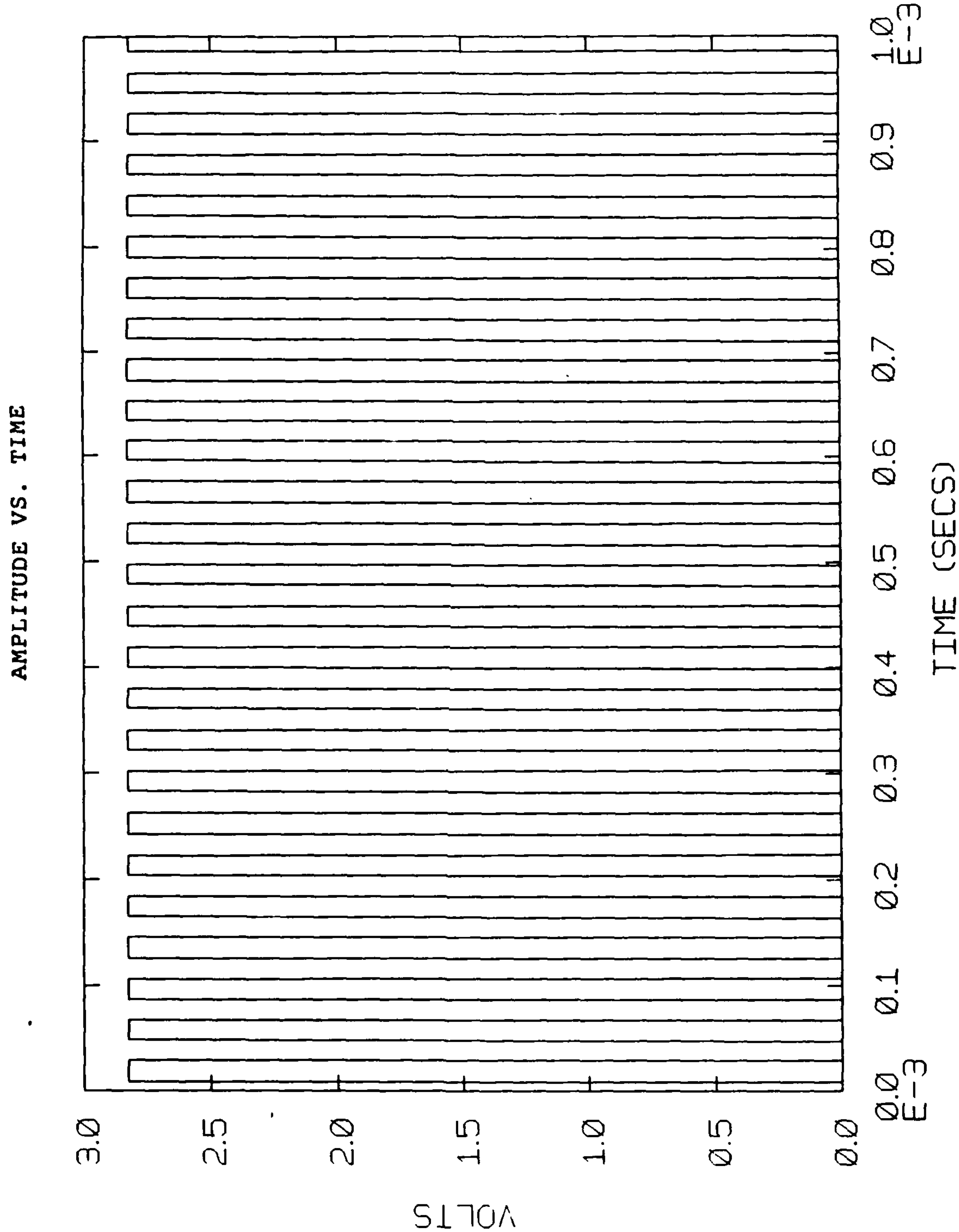

AMPLITUDE VS. TIME
VOLTS
3.0
2.5
2.0
1.5
1.0
0.5
0.0
TIME (SECS)
0.0 E-3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0 E-3

FIGURE 6

SPECTRUM LEVEL VS. FREQUENCY

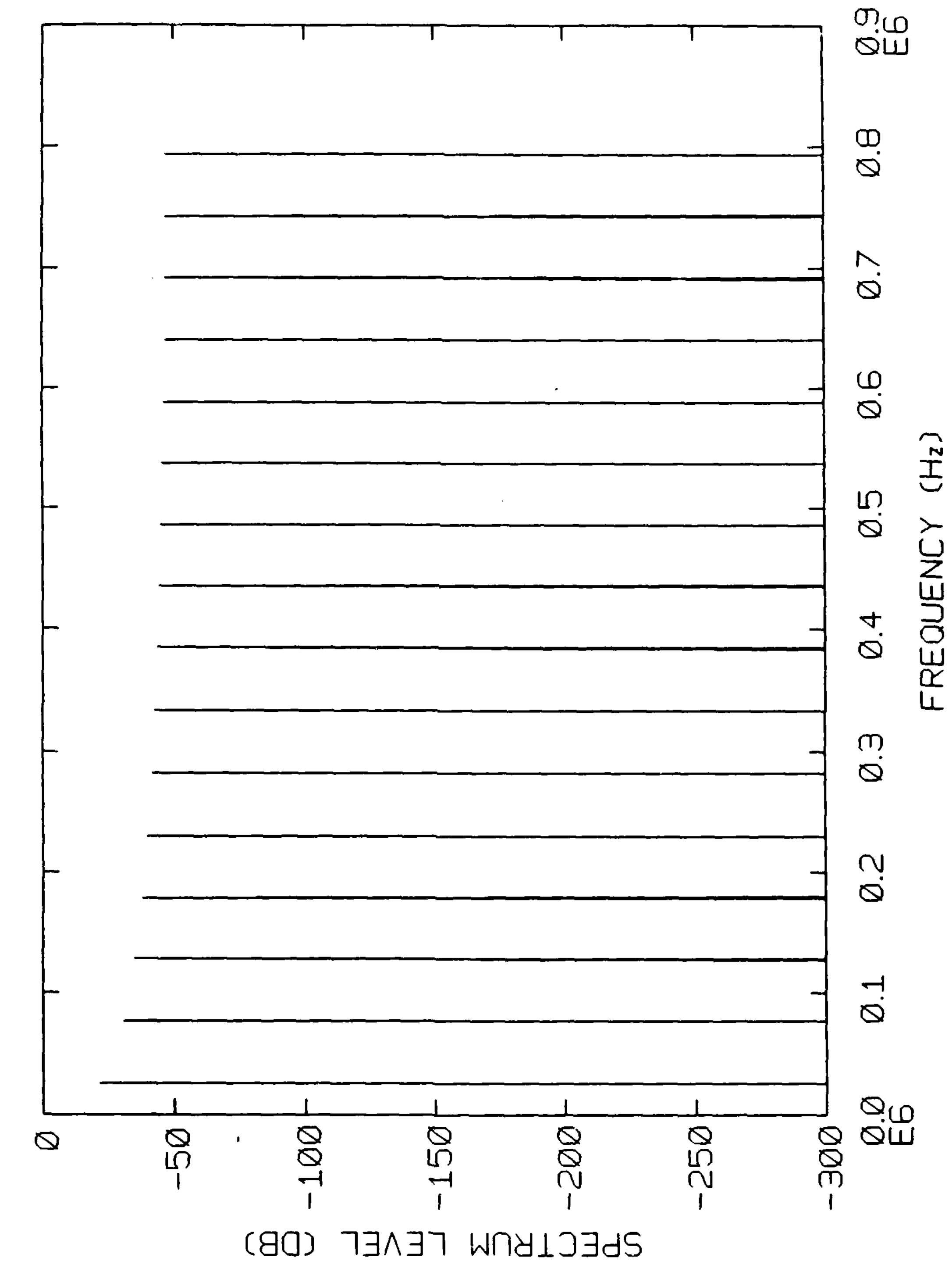
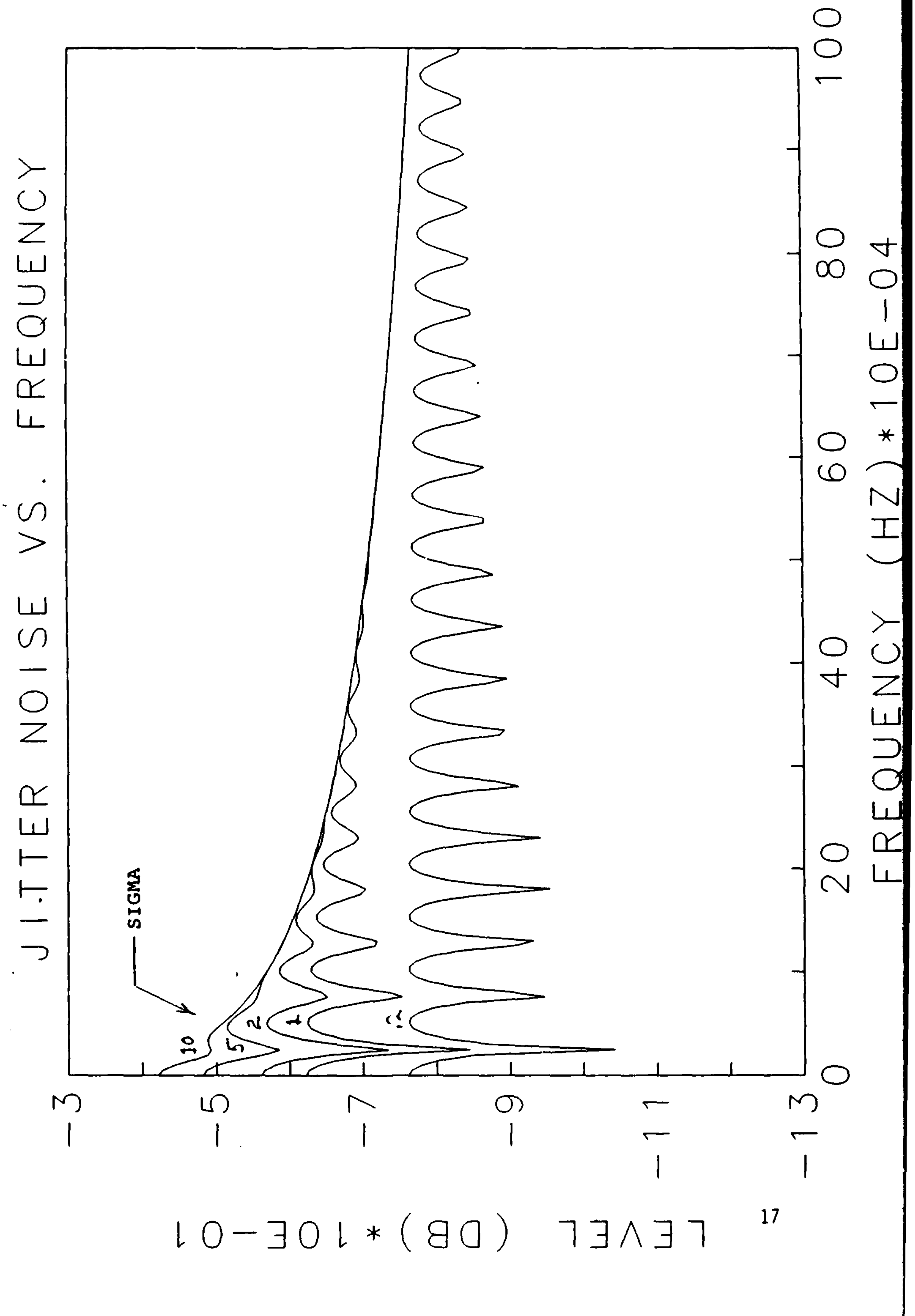
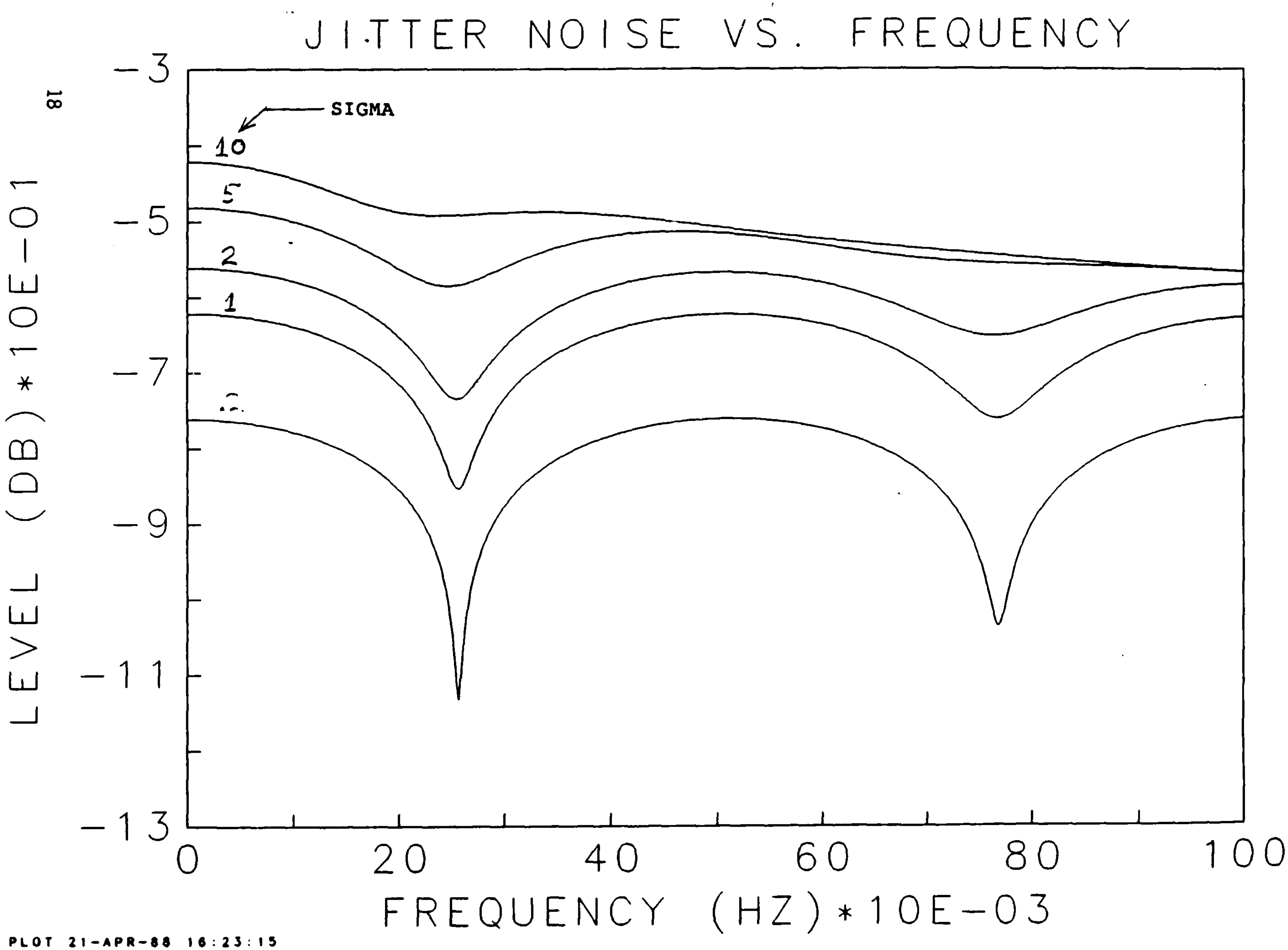


FIGURE 7


JITTER NOISE VS. FREQUENCY
SIGMA
10
5
2
1
.5
-3
-5
-7
-9
-11
-13
LEVEL (DB)*10E-01
0
20
40
60
80
100
FREQUENCY (HZ)*10E-04


JITTER NOISE VS. FREQUENCY
SIGMA
10
5
2
1
-3
-5
-7
-9
-11
-13
0
20
40
60
80
100
LEVEL (DB)*10E-01
FREQUENCY (HZ)*10E-03
PLOT 21-APR-88 16:23:15

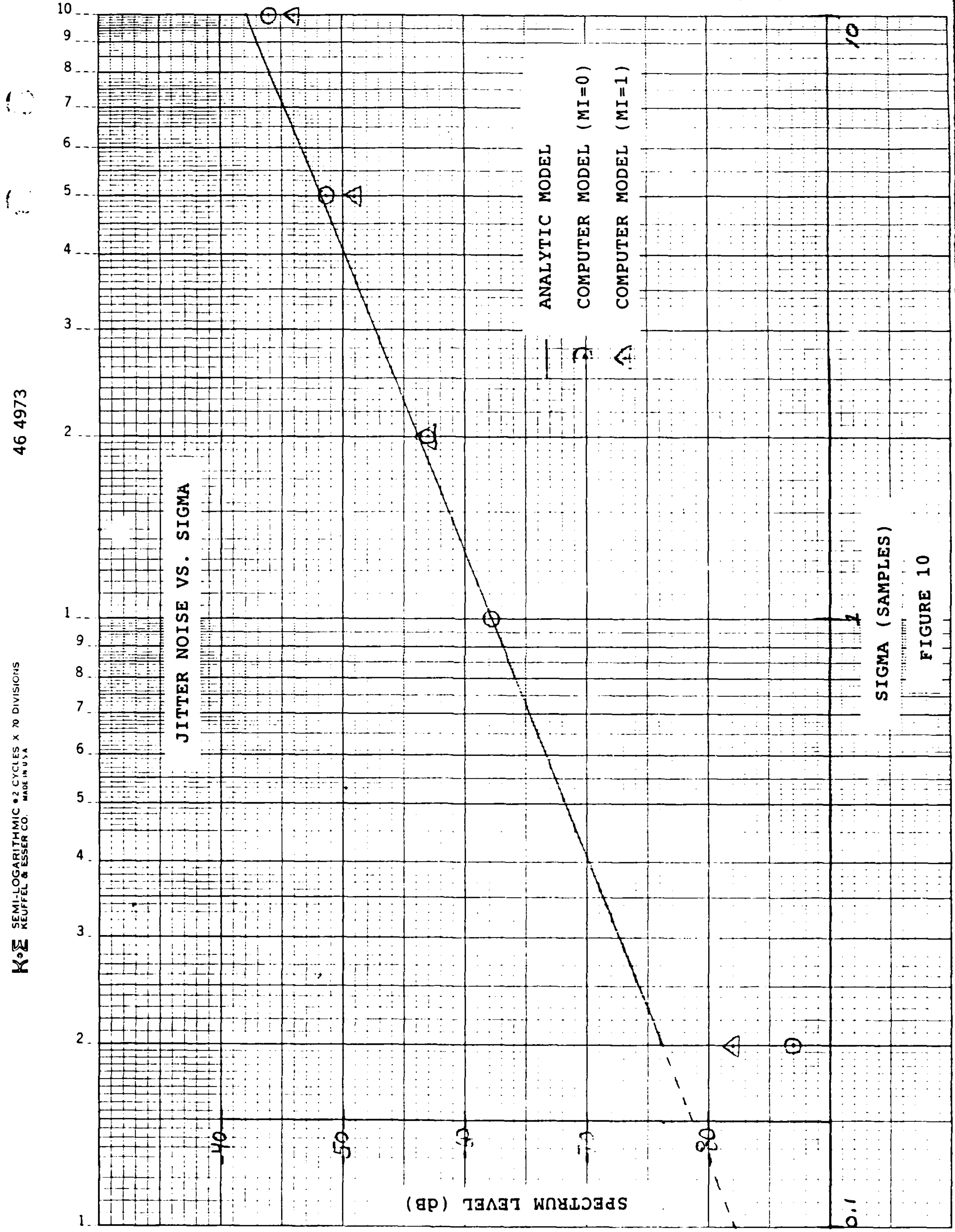

JITTER NOISE VS. SIGMA
ANALYTIC MODEL
COMPUTER MODEL (MI=0)
COMPUTER MODEL (MI=1)
SIGMA (SAMPLES)
SPECTRUM LEVEL (dB)
-40
-50
-60
-70
-80
0.1
1
10
46 4973
K&E SEMI-LOGARITHMIC • 2 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

FIGURE 10

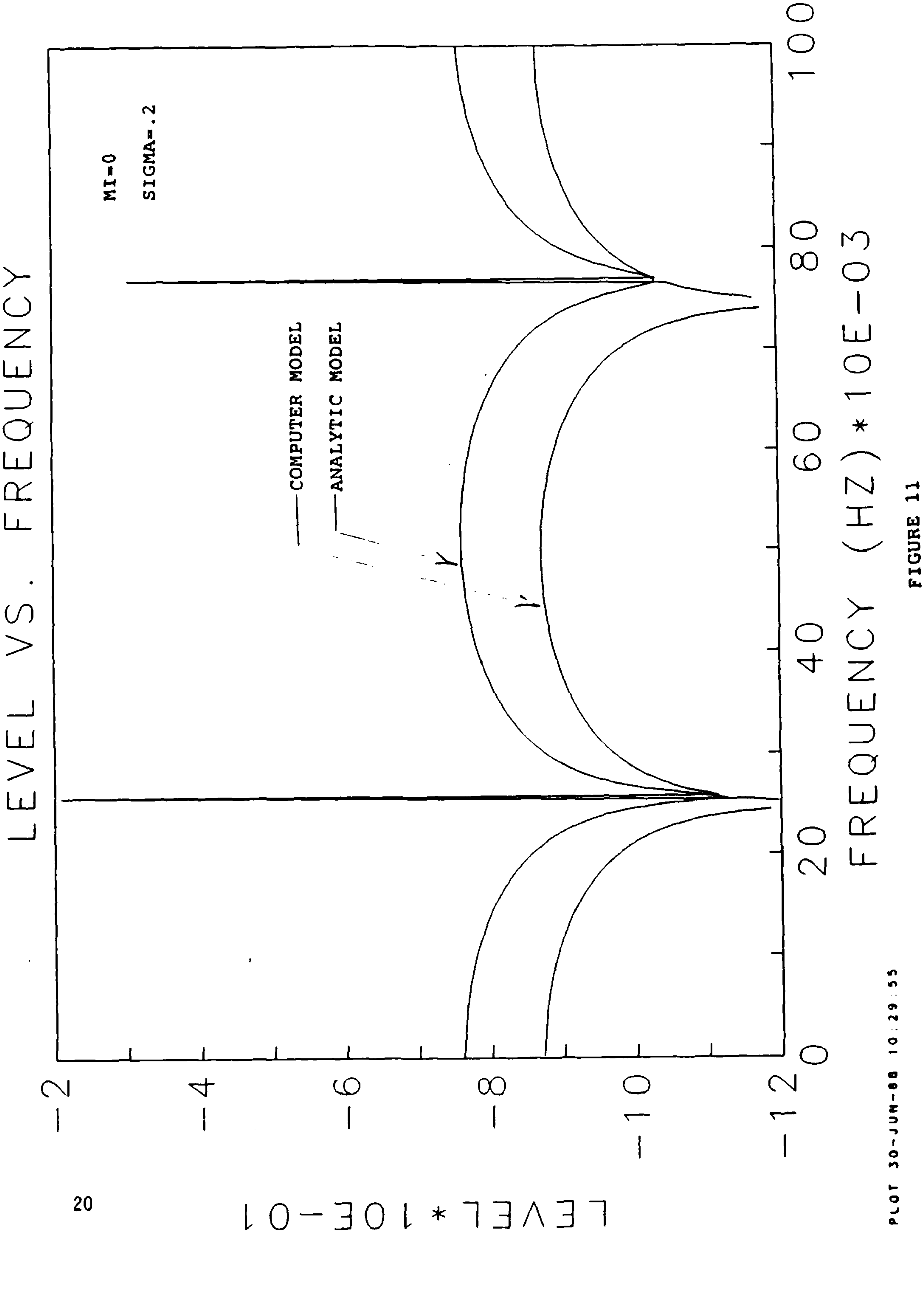

LEVEL VS. FREQUENCY
MI=0
SIGMA=.2
COMPUTER MODEL
ANALYTIC MODEL
0
20
40
60
80
100
FREQUENCY (HZ)*10E-03
-2
-4
-6
-8
-10
-12
LEVEL*10E-01
PLOT 30-JUN-88 10:29:55

FIGURE 11

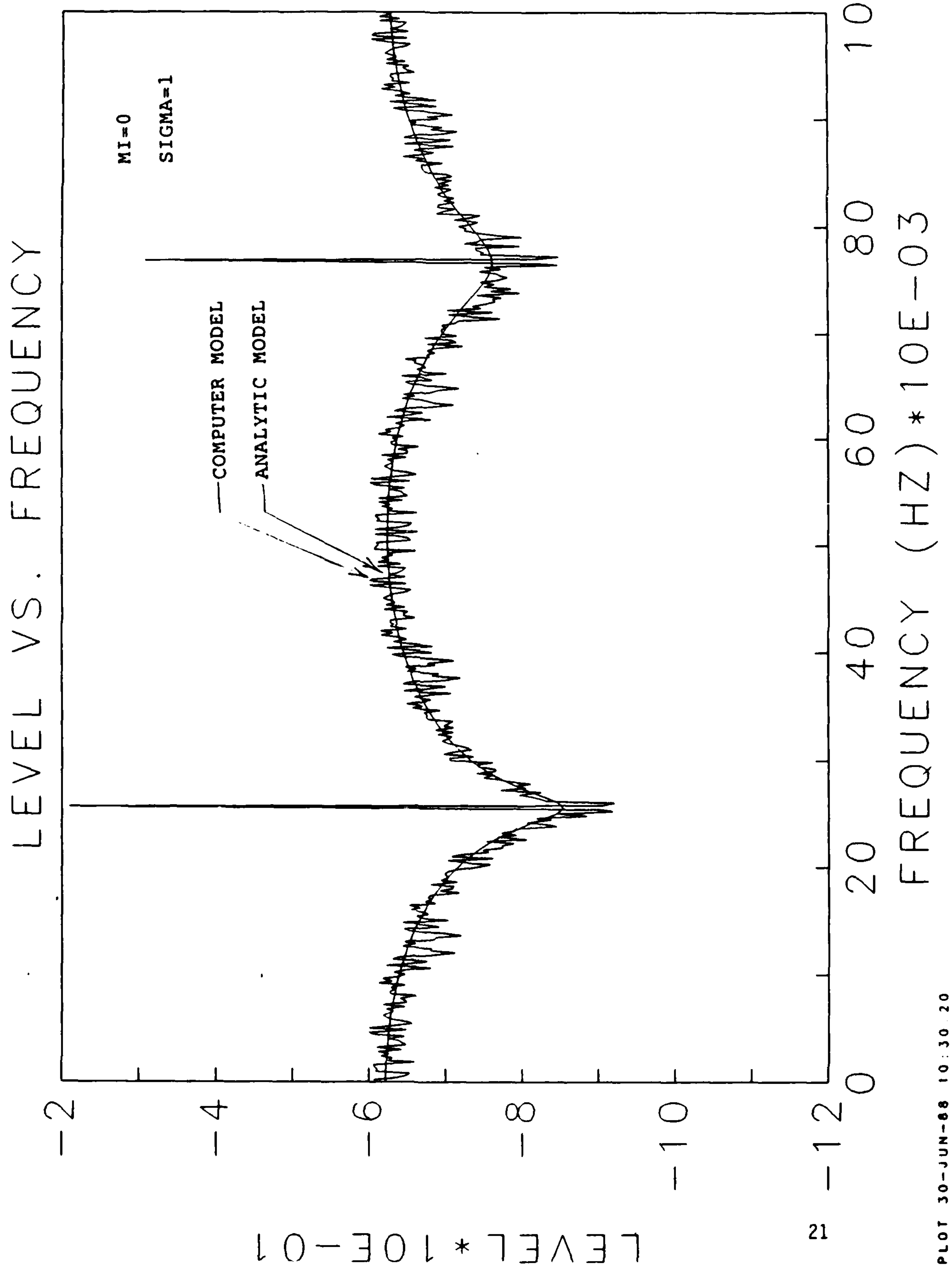

LEVEL VS. FREQUENCY
MI=0
SIGMA=1
COMPUTER MODEL
ANALYTIC MODEL
-2
-4
-6
-8
-10
-12
LEVEL * 10E-01
0
20
40
60
80
100
FREQUENCY (HZ) * 10E-03
PLOT 30-JUN-88 10:30:20

FIGURE 12

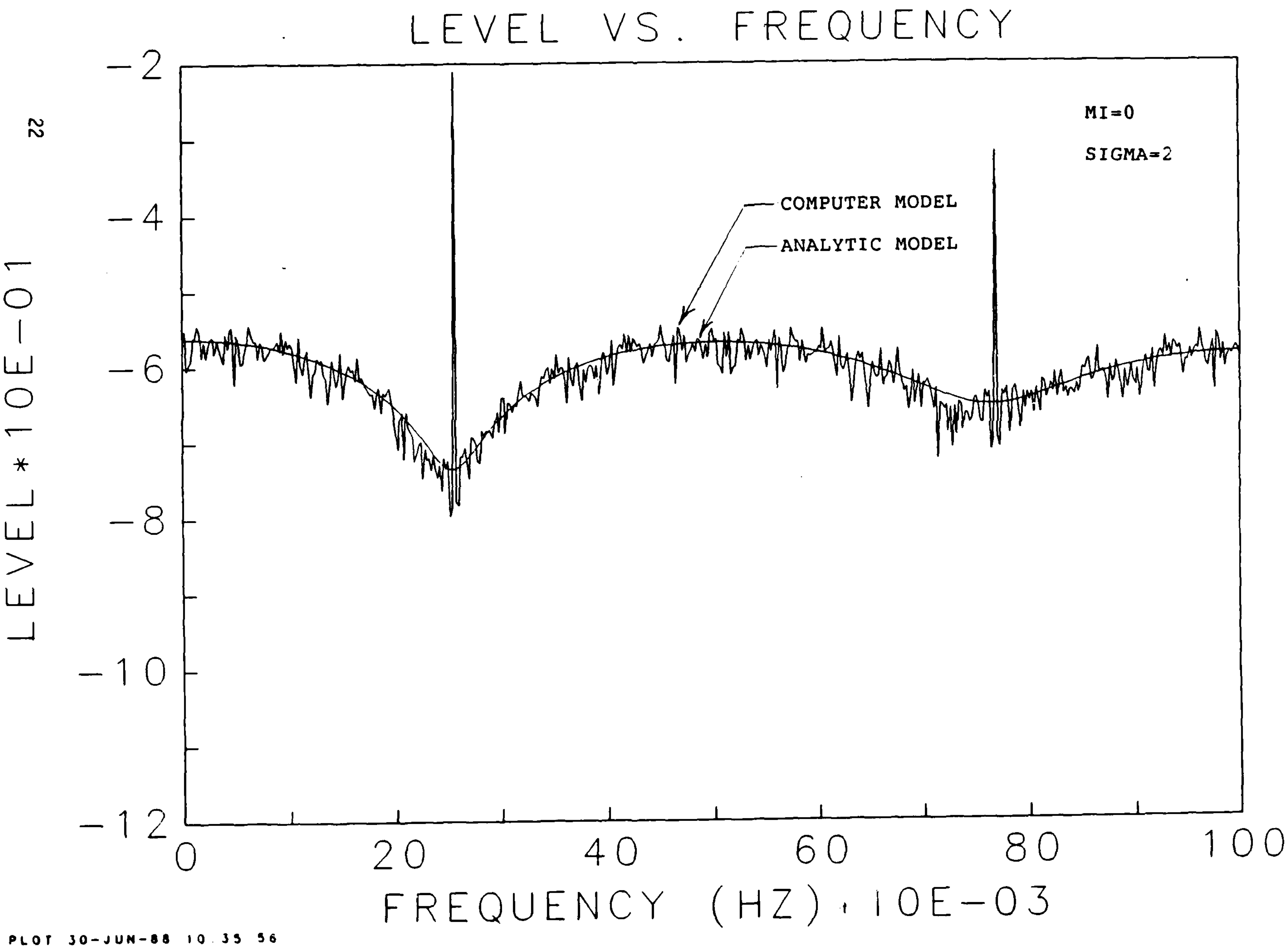
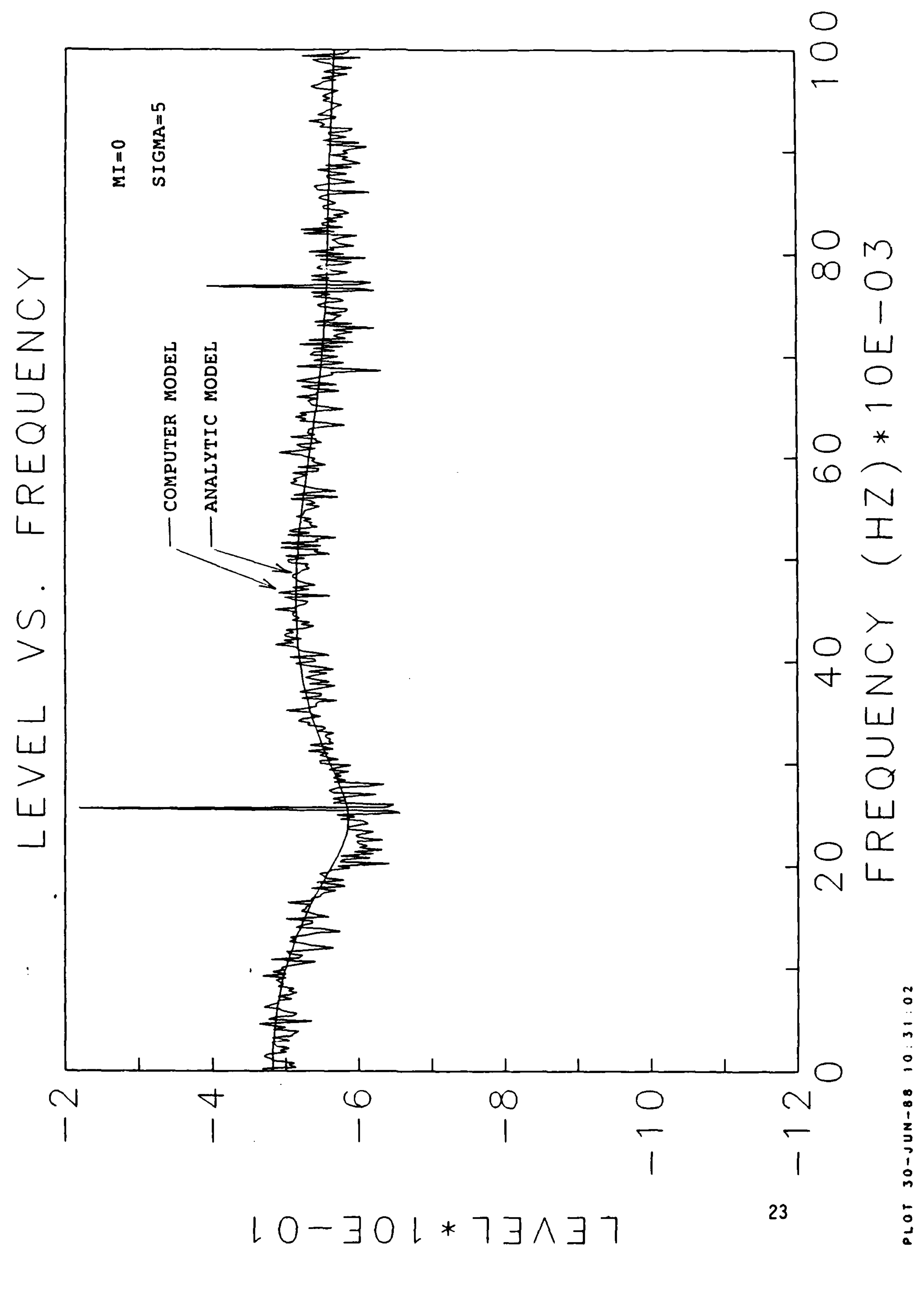

LEVEL VS. FREQUENCY
MI=0
SIGMA=2
COMPUTER MODEL
ANALYTIC MODEL
-2
-4
-6
-8
-10
-12
LEVEL*10E-01
0
20
40
60
80
100
FREQUENCY (HZ) 10E-03
PLOT 30-JUN-88 10 35 56

FIGURE 13



LEVEL VS. FREQUENCY
MI=0
SIGMA=5
COMPUTER MODEL
ANALYTIC MODEL
FREQUENCY (HZ)*10E-03
LEVEL*10E-01
0
20
40
60
80
100
-2
-4
-6
-8
-10
-12
PLOT 30-JUN-88 10:31:02

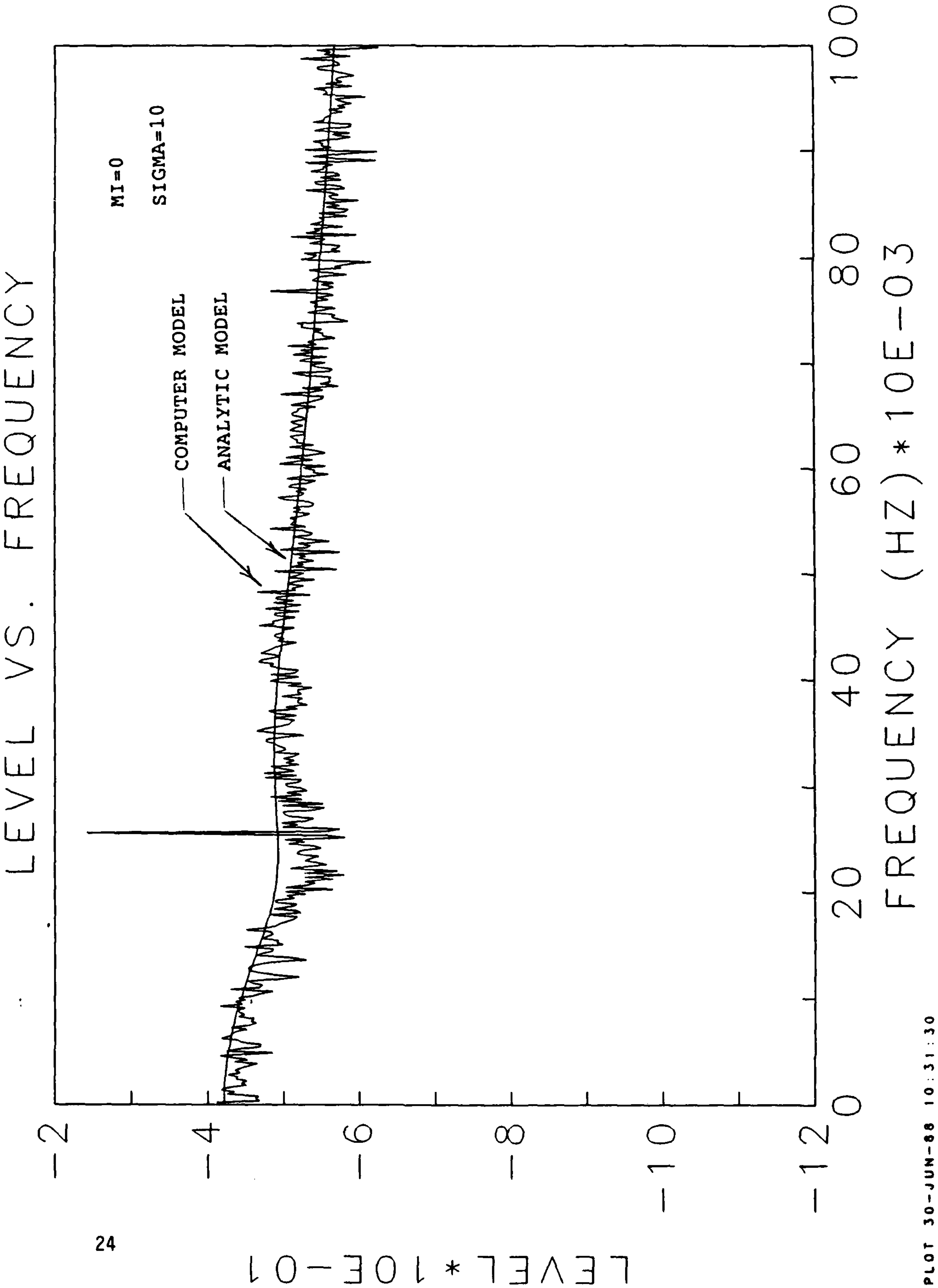
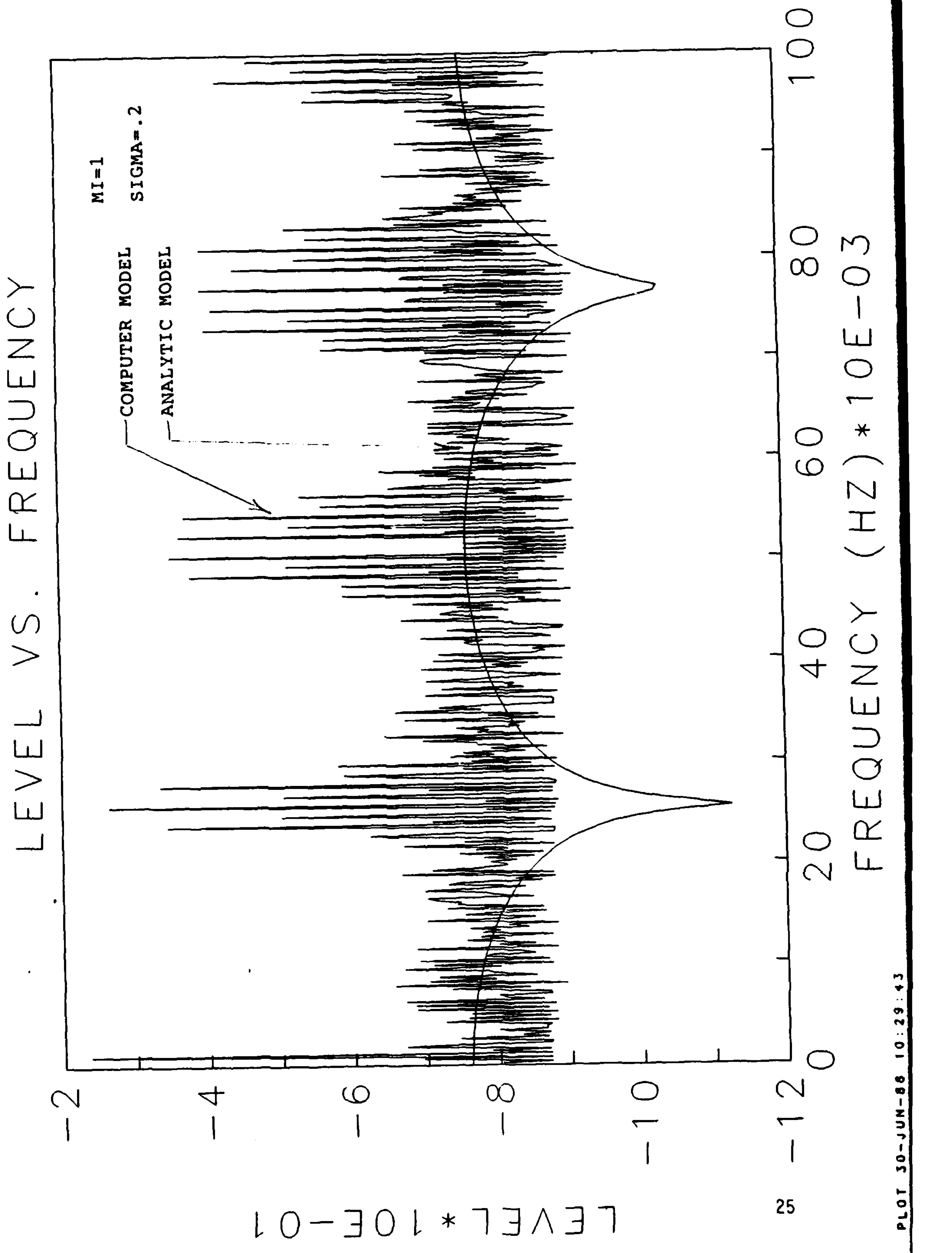

LEVEL VS. FREQUENCY
MI=0
SIGMA=10
COMPUTER MODEL
ANALYTIC MODEL
-2
-4
-6
-8
-10
-12
0
20
40
60
80
100
FREQUENCY (HZ)*10E-03
LEVEL*10E-01
PLOT 30-JUN-88 10:31:30

FIGURE 15


LEVEL VS. FREQUENCY
COMPUTER MODEL
ANALYTIC MODEL
MI=1
SIGMA=.2
FREQUENCY (HZ)*10E-03
LEVEL*10E-01
0
20
40
60
80
100
-2
-4
-6
-8
-10
-12
PLOT 30-JUN-88 10:29:43

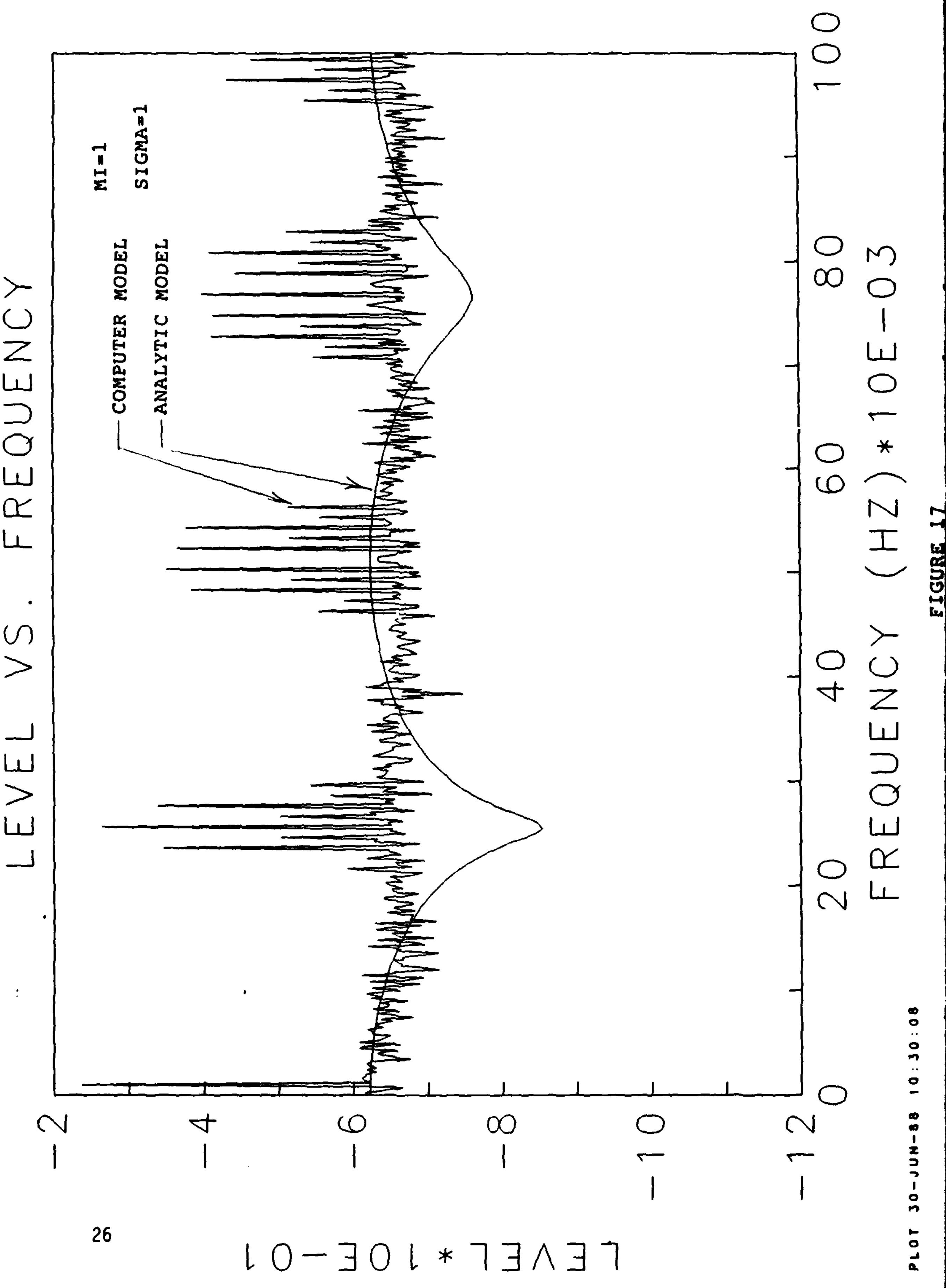

LEVEL VS. FREQUENCY
MI=1
SIGMA=1
COMPUTER MODEL
ANALYTIC MODEL
FREQUENCY (HZ)*10E-03
LEVEL*10E-01
PLOT 30-JUN-88 10:30:08

FIGURE 17

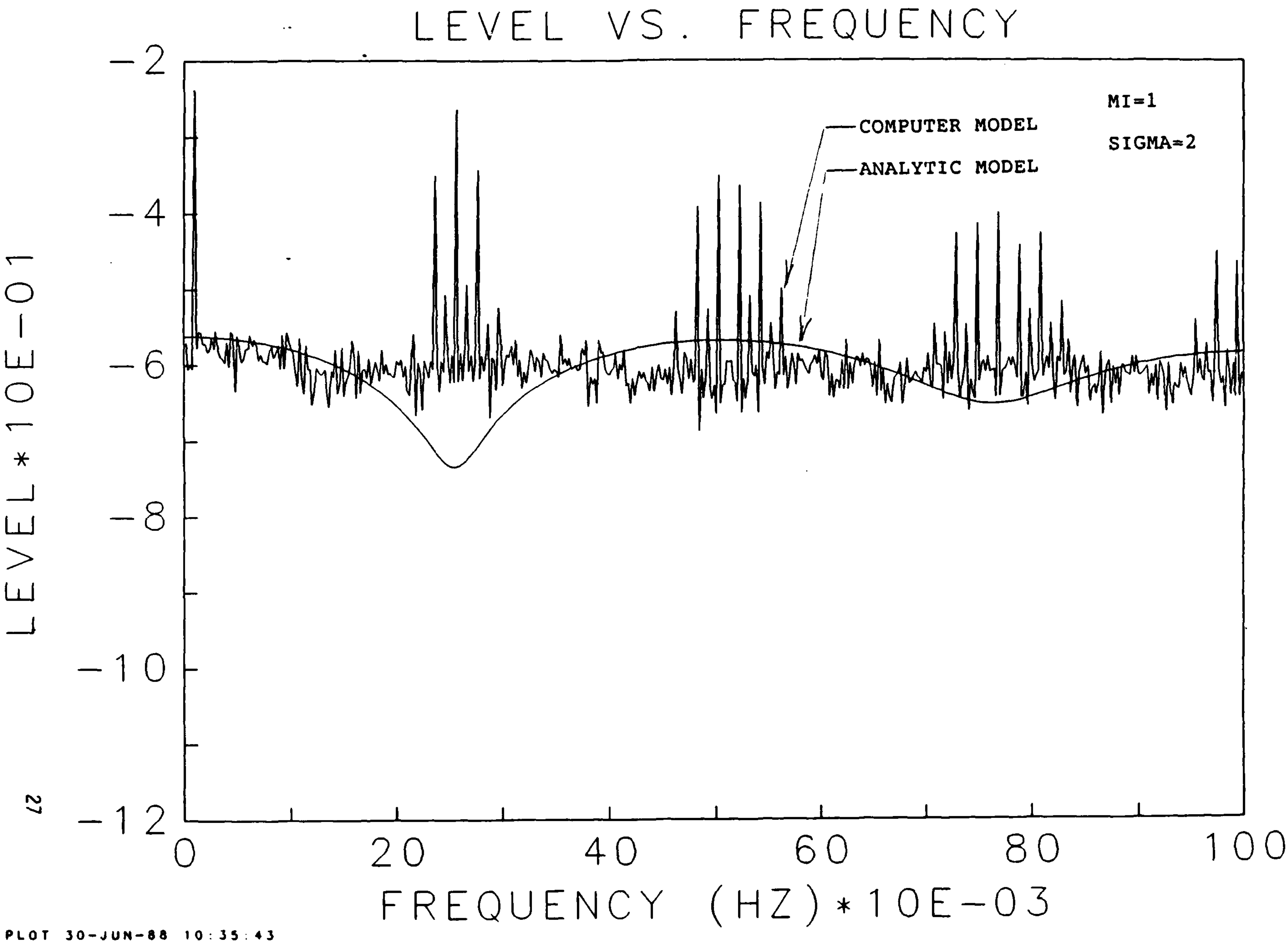

LEVEL VS. FREQUENCY
MI=1
SIGMA=2
COMPUTER MODEL
ANALYTIC MODEL
-2
-4
-6
-8
-10
-12
0
20
40
60
80
100
LEVEL*10E-01
FREQUENCY (HZ)*10E-03
PLOT 30-JUN-88 10:35:43

FIGURE 18

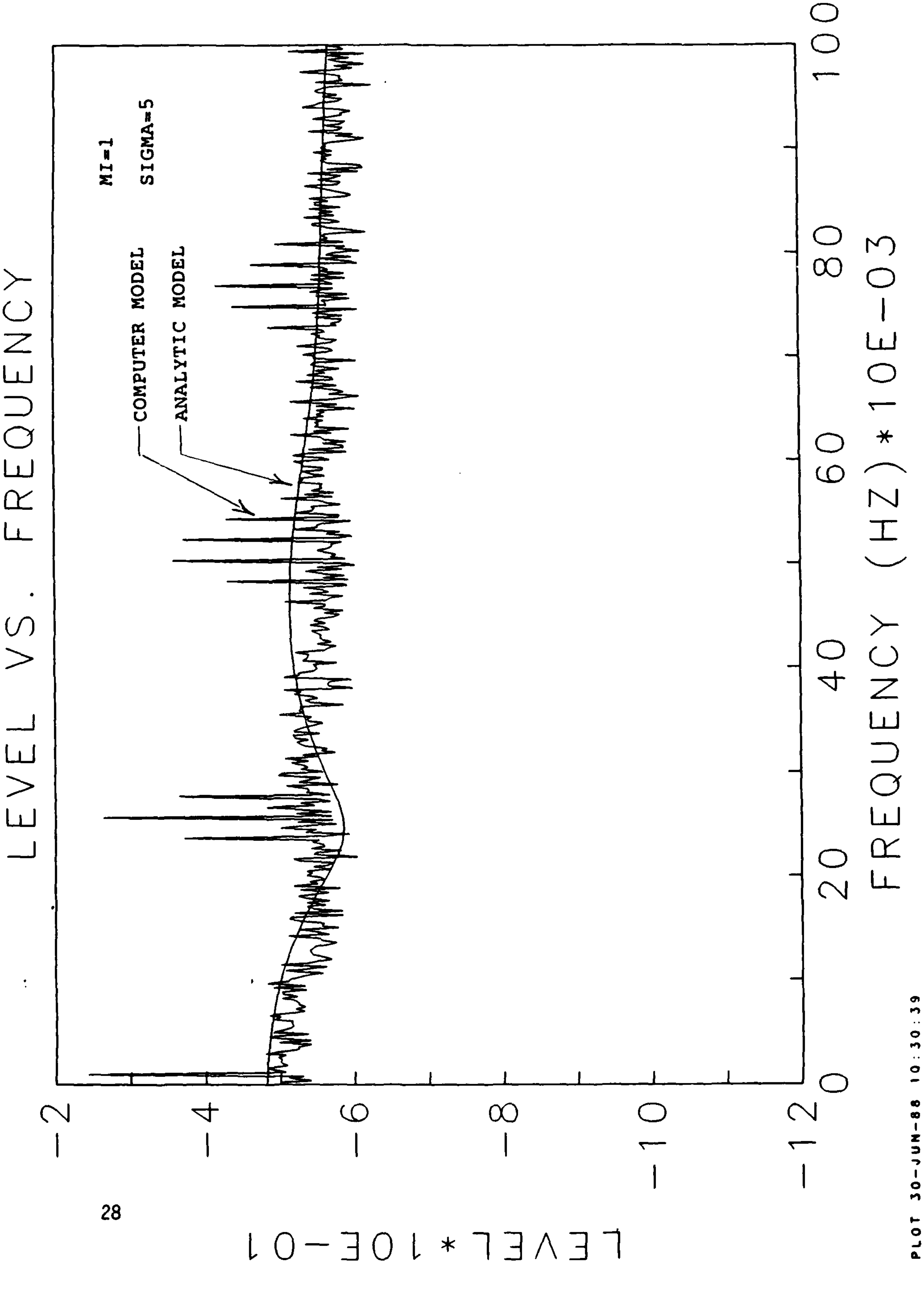
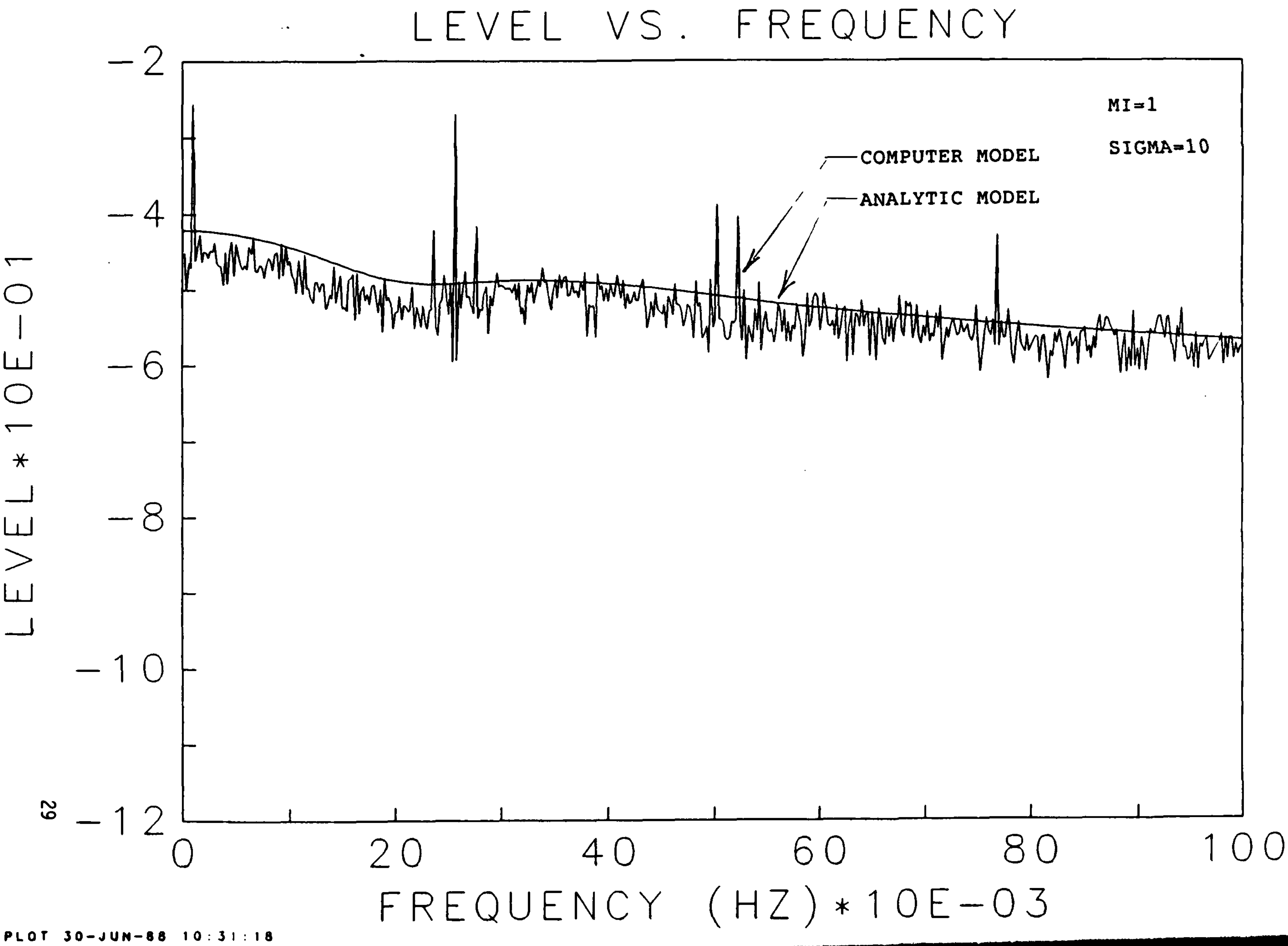


FIGURE 19

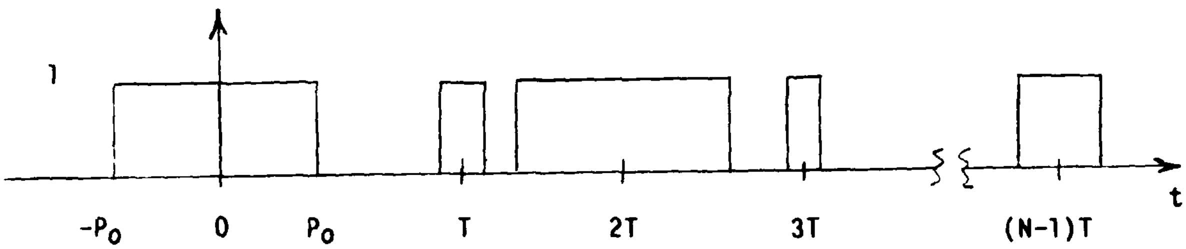
PLOT 30-JUN-88 10:30:39


LEVEL VS. FREQUENCY
MI=1
SIGMA=10
COMPUTER MODEL
ANALYTIC MODEL
-2
-4
-6
-8
-10
-12
LEVEL*10E-01
0
20
40
60
80
100
FREQUENCY (HZ)*10E-03
PLOT 30-JUN-88 10:31:18

APPENDIX I

Consider the following sequence of pulses where

$f_w = 1/T \rightarrow$ switching frequency



Assume the pulse width of the n^{th} pulse is given by

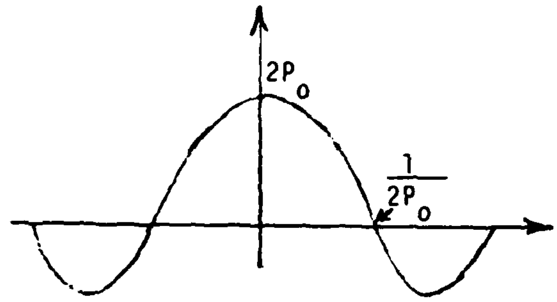
$$P = P + \delta_n$$

where δ_n is a Gaussian r.v. with density function

$$p_\delta = \frac{1}{\sqrt{2\pi}\,\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2}$$

The Fourier transform of the pulse at the origin is

$$X_o(f) = \frac{\sin 2\pi f\, P_o}{\pi f}$$



The Fourier transform of a sequence on N pulses is given by

$$X(f) = \sum_{n=0}^{N-1} e^{-j2\pi fnT} \frac{\sin 2\pi f\, P_n}{\pi f}$$

and

$$X(f)X^*(f) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \left\{ \cos[2\pi fT(n-m)] \frac{\sin 2\pi\, f\, P_n \sin 2\pi f\, P_m}{\pi^2\, f^2} \right\}$$

The one sided power spectrum, corrected for bandwidth and signal amplitude is given by

$$S(f) = \frac{2\, A^2\, f_w}{N}\; E \left\{ X(f)\; X^*(f) \right\}$$

For the case where we have an unmodulated class A-D signal (square wave)

$$2P = T/2 = 1/(2f_w)$$

and after considerable manipulation we can write the power spectrum for the case where $f \neq f_W$ as

$$S(f) = \frac{A^2 f_W}{\pi^2 f^2} \left\{ \left[1 - e^{-(2\pi f \sigma_\delta)^2} \right] \left[1 + \cos\left(\frac{\pi f}{f_W}\right) e^{-(2\pi f \sigma_\delta)^2} \right] \right\}$$

Using approximation formula valid for low frequency ($f \ll f_W$) the power spectrum can be written

$$S(f) \approx 8 A^2 f_W \sigma_\delta^2$$

The units of σ_δ are time (seconds). The main text, however, has units of standard deviation in terms of the number of samples of the sampling frequency. With K = 64 digital samples per cycle of switching frequency we can also write

$$S(f) \approx \frac{8 A^2 \sigma_s^2}{K^2 f_W}$$

where (in the text) σ_s takes on the values .2, 1, 2, 5 and 10.

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R. J. Nielsen
TM No. 881110
5 August 1988

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